Equivariant homotopy theory learning seminar

Spring 2024

The goal for this seminar is to learn the basics of equivariant (stable) homotopy theory, including some of the modern terminology and techniques in the field. To facilitate this, we will mostly follow **Andrew Blumberg's notes (Blum)**, supplemented by the following (and other) sources for additional topics and alternative perspectives:

- Hill–Hopkins–Ravenel, *On the non-existence of elements of Kervaire invariant one* (HHR).
- Hill–Hopkins–Ravenel, *Equivariant stable homotopy theory and the Kervaire invariant problem* (HHRb).
- May, *Equivariant homotopy and cohomology theory* (Alaska).
- Schwede, *Lectures on equivariant stable homotopy theory* (Schwede).

Format: Weekly 1 hour talks from participants.

Meeting information: Tuesday, 9:45am, Altgeld 141 (backup location: Noyes 164).

Week 1 (1/30). Equivariant homotopy theory of spaces.

- Include: G-spaces and equivariant maps, Homotopies of equivariant maps, Homotopy groups, G-CW complexes, Elmendorf's theorem.
- Blum: 1.1-1.3; Alaska: I.1-I.3, V.3-V.4.

Week 2 (2/6). Equivariant cohomology.

- Include: Bredon cohomology, Borel cohomology, Representation spheres and $\mathrm{RO}(G)$-gradings, Brown representability.
- Blum: 1.4, 1.5, 3.3; Alaska: Chapter IV.

Week 3 (2/13). G-spectra.

- Include: theoretical aspects, required properties of G-spectra, orthogonal G-spectra, spectra with G-action, comparisons.
- Blum: Chapter 2; Schwede: Sections 2-3; HHR: Section 2; HHRb: Part II, Chapter 9.

Week 4 (2/20). G-spectra continued.

- Include: Finish the material from the previous week. Wirthmueller Iso. Tom Dieck splitting. Invertibility of representation spheres.
- Blum: Chapter 2, specifically 2.6 and 2.7; Schwede: Sections 2-3; HHR: Section 2; HHRb: Part II, Chapter 9.

Week 5 (2/27). Equivariant algebra for G-spectra.

- Include: Mackey functors, Burnside category and ring, Mackey Functor valued cohomology, (maybe: spectral Mackey functors as G-spectra).
- Blum: Chapter 3; HHR: Section 3.

- Maybe: Barwick, *Spectral Mackey functors and equivariant algebraic K-theory*.

Week 6 (3/5). Spectral sequences.

- Include: Homotopy fixed point spectral SS, homotopy orbit SS, slice SS. Work/sketch through your choice of computation.
- HHR: Section 4; Blum: Chapter 5; HHRb: Part III, Chapter 11. Heard Stojanoska, *K-Theory, Reality, and Duality*: Chapter 5

Spring break

Week 7 (3/19). Examples of G-spectra.

- Potential topics: Equivariant K-theory, Borel equivariance for any spectrum, Atiyah–Segal completion, KR-theory, Real bordism, equivariant Eilenberg–Mac Lane spectra, Morava E-theory, Segal conjecture, equivariant cobordism.
- HHR: Section 5; HHRb: Part III, Chapter 12; Schwede: 2.1; Hahn–Shi; any more of speaker's choice.

Week 8 (3/26). Multiplicative structures.

- Include: Tambara functors, norms, N_∞-structures, orthogonal (commutative) ring G-spectra, examples (Eilenberg–Mac Lane, $\mathrm{MU}_{\mathbb{R}}$, KU_G, KR), important results?
- Blum: Chapter 4; HHRb: Part II, Chapter 10; Blumberg–Hill.

Week 9 (4/2). Real oriented homotopy theory.

- Include: Real bordism, Real orientations (multiple perspectives), KR, E_n, HHR-theorem.
- HHRb: Part III; HHR; Hahn–Shi; Hu–Kriz

Week 10 (4/9). Global homotopy theory.

- Schwede, *Global homotopy theory*.

Options for additional topics:

- Equivariant algebraic K-theory (Merling, Barwick)
- Equivariant elliptic cohomology
- Examples from geometry and physics
- Freed–Hopkins–Teleman theorem
- Equivariant bundle theory (Alaska)
- Applications in chromatic homotopy theory
- Equivariant motivic homotopy theory