On Ryser's conjecture and strengthenings thereof

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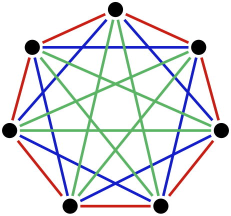
joint work with Hannah Sheats, Teresa Flaitz, Cody Newman,
Delaney Aydel, Grace McCourt, and Louis DeBiasio

March 22, 2019



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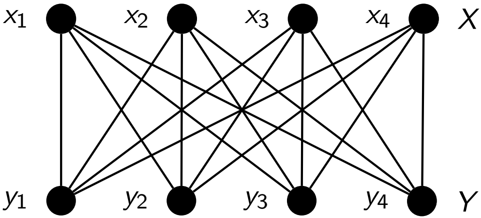
Definitions

- A hypergraph is r-**partite** if there is a partition of its vertices into r classes such that each edge intersects every class in at most one vertex.
- A hypergraph is r-**uniform** if every edge contains exactly r vertices.
- A **matching** M is a set of pairwise disjoint edges.
  $\nu(H) := |\text{maximum matching in } H|$.
- A **vertex cover** is a set of vertices that intersects every edge.
  $\tau(H) := |\text{minimum vertex cover of } H|$.

Note: $\nu(H) \leq \tau(H)$ for all hypergraphs H

Example: Complete Bipartite Graph $K_{4,4}$



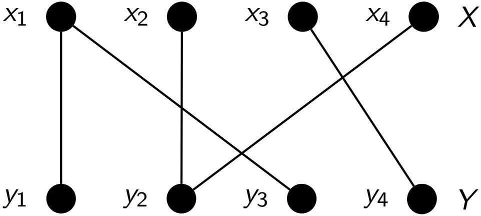
König's Theorem

Theorem (König)

For every bipartite graph B, $\tau(B) \le \nu(B)$.

Example

Maximum matching?
Minimum vertex cover?



König's Theorem

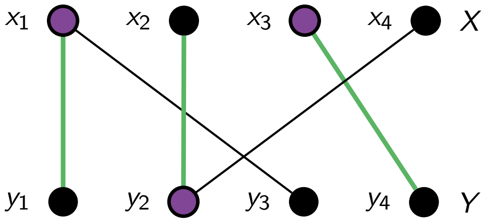
Note: $\nu(H) \leq \tau(H)$ for all hypergraphs H

Theorem (König 1931)

For every bipartite graph B, $\tau(B) \leq \nu(B)$.

Example

Maximum matching: $M = \{x_1y_1, x_2y_2, x_3y_4\}$.
Minimum vertex cover: $T = \{x_1, y_2, x_3\}$.



Back to the Beginning: (0,1)-Matrices

$$
\begin{array}{c c}
 & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\
\begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\end{array}
$$

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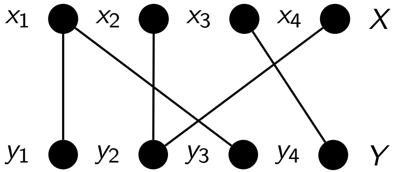

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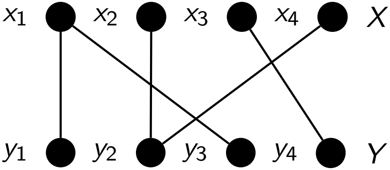

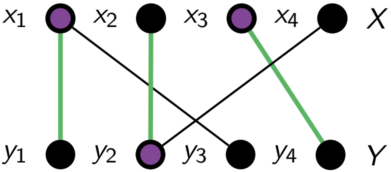

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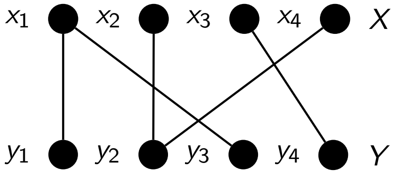

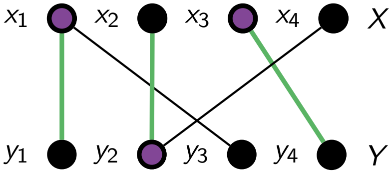

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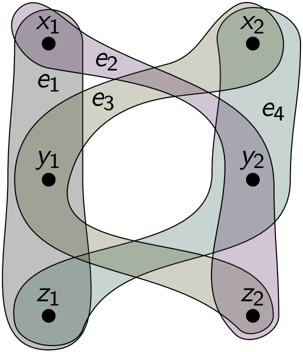
Ryser's Conjecture

Conjecture (Ryser 70's)

For every r-partite r-uniform hypergraph H, $\tau(H) \leq (r-1)\nu(H)$.

Example for $r = 3$

Maximum matching?
Minimum vertex cover?



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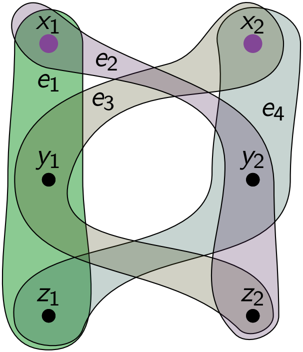
Minimum vertex cover: $T = \{x_1, x_2\}$.



Ryser's Conjecture is Best Possible: Projective Planes

Theorem (Tuza 1983)

When $r - 1$ is a prime power then the inequality in Ryser's conjecture is best possible.

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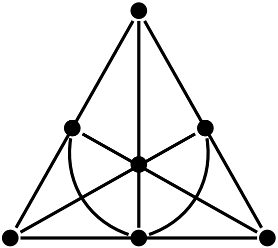

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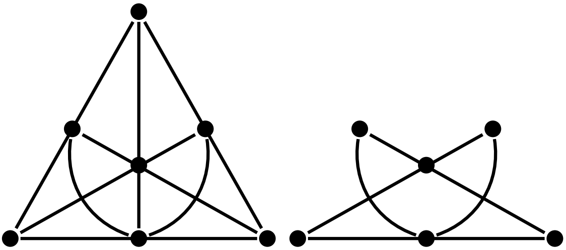

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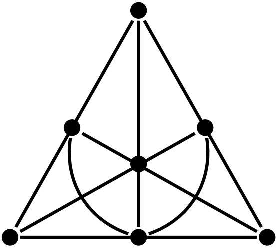
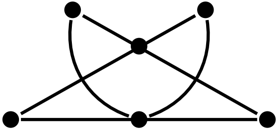
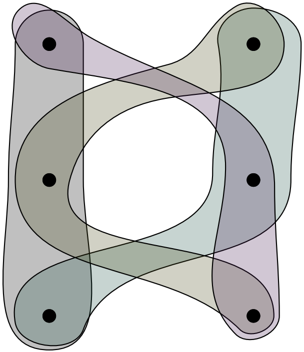
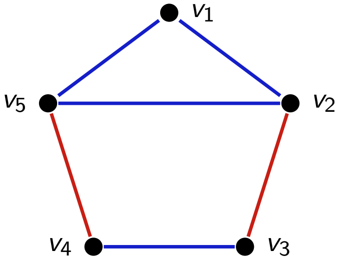






Edge-Colorings: Preliminaries

- **connected**, **component**
- **r-coloring**
- **component cover**
  $tc_r(G) := \max |\text{minimum component cover}|$
- **independent**
  $\alpha(G) := |\text{maximum independent set}|$



Hypergraphs $\rightarrow$ Egde-Colorings

H: r-partite r-uniform hypergraph $\rightarrow$ G: r-colored graph

- edge in H $\rightarrow$ vertex in G
- intersection between edges in H $\rightarrow$ edge between vertices in G
- partition class in H $\rightarrow$ color in G

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Using this correspondence, we obtain the following:

- matching in $H \rightarrow$ independent set in G
- vertex in $H \rightarrow$ monochromatic component of G
- vertex cover in $H \rightarrow$ component cover in G
- $\tau(H) \rightarrow tc_r(G); \quad \nu(H) \rightarrow \alpha(G)$

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Let's rephrase König's Theorem in terms of G:

Theorem

For every bipartite graph B,
$\tau(B) \leq \nu(B)$.

$\rightarrow$

Theorem

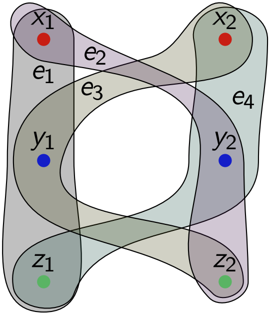
For every graph G,
$tc_2(G) \leq \alpha(G)$.

Re-Ryser's Conjecture: Edge-Colorings

Conjecture

For any graph G, $tc_r(G) \leq (r-1)\alpha(G)$.

Example for $r = 3$



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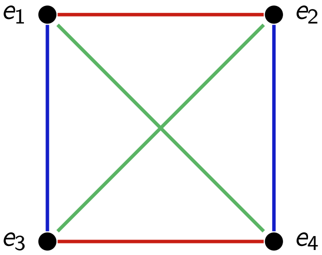

Ryser's Conjecture: What is Known?

Conjectured values for $tc_r(G)$

α \ r	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25
6	$\downarrow$	$\downarrow$	18	24	30

- $r = 2$: König (1931)
- $r = 3$: Aharoni (2001)
- $r = 4, 5; \alpha = 1$: Tuza (1978)

Why Edge-Colored Graphs? Interesting Variants

- **Bounded Diameter**
  - The **distance** between two vertices in a graph is the length of the shortest path between them.
  - The **diameter** of a graph is the maximum distance between pairs of its vertices.
  - We say $dc_r(G) \leq k$ if for any r-coloring of G, there is a set of at most k monochromatic subgraphs of bounded diameter (independent of $|V|$) that cover the vertices of G.
  - **Question:** $dc_r(G) \leq (r-1)\alpha(G)$?
- **Partitions**
  - We say $tp_r(G) \leq k$ if for any r-coloring of G, there is a set of at most k monochromatic connected subgraphs that partition the vertices of G.
  - **Question:** $tp_r(G) \leq (r-1)\alpha(G)$?
- **Special Subgraphs**
  - Paths, cycles, stars, brooms, etc.
- **Edge-Colored Hypergraphs**

Variants: What is Known?

Bounded Diameter:

Conjectured values for $dc_r(G)$

α \ r	**2**	**3**	**4**	**5**
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5	5	10	15	20

- $r = 2; \alpha = 1$: Folklore
- $r = 3; \alpha = 1$: Milićević (2015)
- $r = 4; \alpha = 1$: Milićević (2017)
- $r = 2; \alpha = 2$: SUMSRI 2018

Ryser's Conjecture: $r = 4$

Theorem (Tuza 1978/SUMSRI 2018)

Let K_n be 4-colored. Given any two colors, there exists a component cover of size at most three that either consists of only those two colors or only the other two.

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Take closure.

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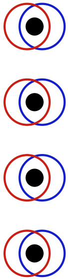

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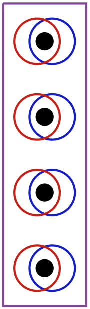

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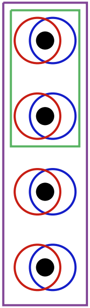

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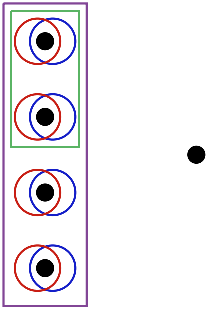

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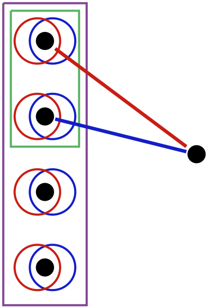

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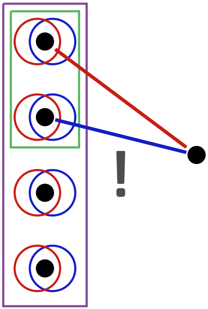

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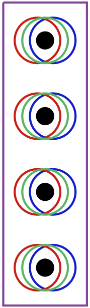

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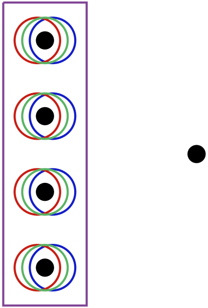

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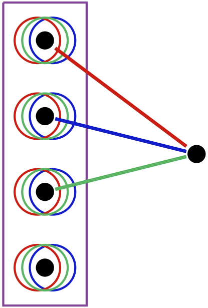

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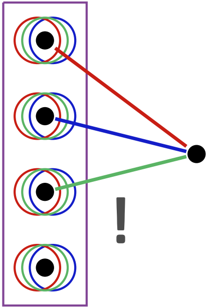

Ryser's Conjecture: Can we say more?

Theorem (SUMSRI 2018)

Let K be a complete 4-partite graph with $V(G) = V_1 \cup V_2 \cup V_3 \cup V_4$. Then $tc_3(K) \leq 3$.

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Let K be a 4-colored complete graph. If S is any subset of the four colors, then there exists a component cover of size at most three that either consists of only those colors in S or only colors not in S.

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Theorem (Tuza 1978/SUMSRI 2018)

Let K_n be 5-colored. Given any two colors, there exists a component cover of size at most four that either consists of only those two colors or only the other three.

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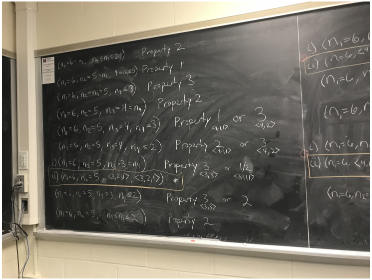
Inquiry

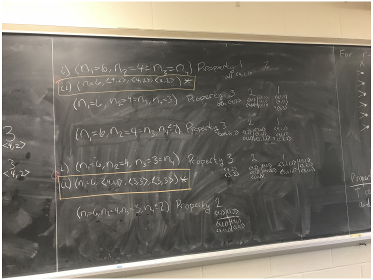
$tc_6(K_n) \leq 5$?

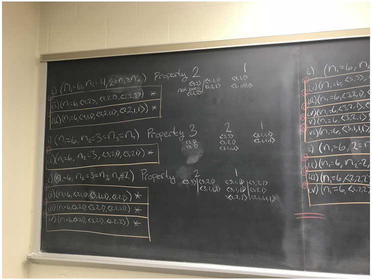
Inquiry

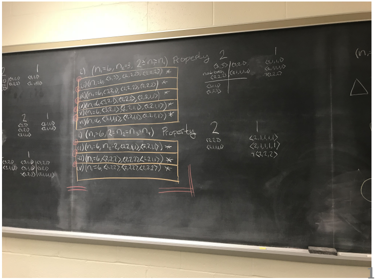
$tc_4(G) \leq 6$ for $\alpha(G) = 2$?

Ryser's Conjecture: $r = 6$









Ryser's Conjecture: $r = 6$

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For any graph G, either G has diameter at most 3 or its complement has diameter at most 2.

Bounded Diameter: $r = 2, \alpha = 1$

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For any 2-coloring of $G = K_n$, either G has diameter at most 3 in one color or diameter at most 2 in the other color.

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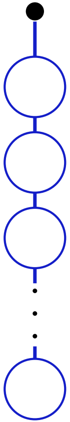

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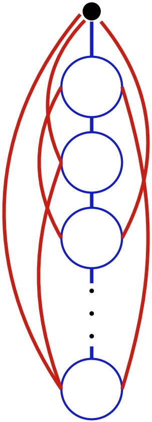

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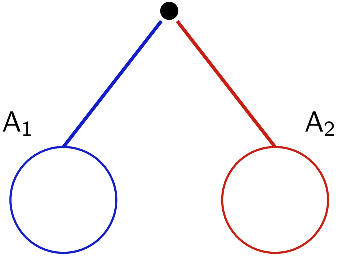

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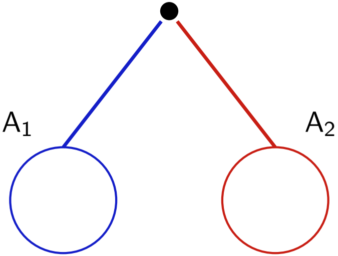

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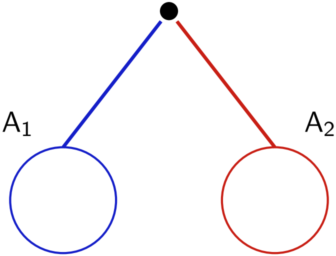

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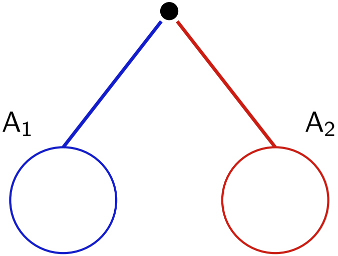

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In general, let $B_{ij} = \{v \in A_i | v \text{ does not share any j-color edges with } A_j\}$.

Bounded Diameter and Fixed Point Theorems

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Lemma (Milićević 2015)

In every 3-coloring of K_n there exists a set of at most two monochromatic subgraphs of diameter at most 8 that cover the vertices of K_n.

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Theorem (Milićević 2017)

In every 4-coloring of K_n there exists a set of at most three monochromatic subgraphs of diameter at most 80 that cover the vertices of K_n.

Bounded Diameter: Improvements

Theorem (SUMSRI 2018)

In every 3-coloring of K_n there exists a set of at most two monochromatic subgraphs of diameter at most 4 that cover the vertices of K_n.

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Theorem (SUMSRI 2018)

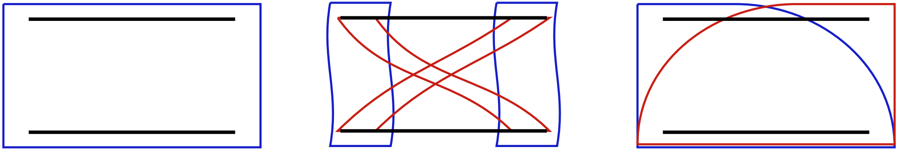
In every 4-coloring of K_n there exists a set of at most three monochromatic subgraphs of diameter at most 6 that cover the vertices of K_n.

Bounded Diameter: Improvements

Lemma (Milićević 2017/SUMSRI 2018)

In any 2-coloring of $K_{n,m}$, one of the following properties holds:

- *There is a color which has diameter at most 6.*
- *There are exactly two components of each color, all of which are non-trivial and diameter 2*
- *For each color, there exists a vertex that is not incident with that color, which gives two components each of diameter at most 3.*



Bounded Diameter: New Results

Theorem (SUMSRI 2018)

In every 3-coloring of $K_{n,m}$ there exists a set of at most four monochromatic subgraphs of diameter at most 6 that cover the vertices of $K_{n,m}$.

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Let G be a graph with $\alpha(G) = 2$. In every 2-coloring of G there exists a set of at most two monochromatic subgraphs of diameter at most 6 that cover the vertices of G.

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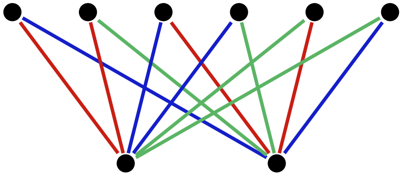
Example A 3-coloring of $K_{6,2}$ that cannot be covered with a set of at most three monochromatic components.



Bounded Diameter: How good can we do?

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Does there exist a fixed integer δ such that any r-coloring of K_n can be covered by $r-1$ monochromatic components with diameters at most δ?

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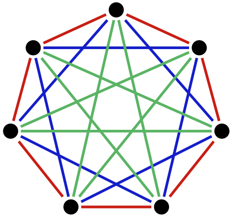
Does there exist a fixed integer δ such that any r-coloring of K_n can be covered by $r-1$ monochromatic components with diameters at most δ?

If such a δ exists, we currently don't have any evidence that would require it to be larger than 3!

Example A 3-coloring of K_7 that cannot be covered with a set of at most two monochromatic subgraphs of diameter at most 2.

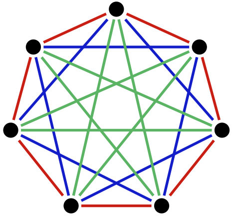


Open Problems: Upper Bounds

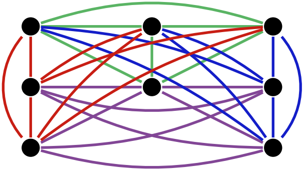
- **Ryser's Conjecture ($r = 6, \alpha = 1$):** Given a 6-coloring of K_n, does there exist a set of at most five monochromatic components that cover the vertices of K_n?
- **Ryser's Conjecture ($r = 4, \alpha = 2$):** Let $\alpha(G) = 2$. Given a 4-coloring of G, does there exist a set of at most six monochromatic components that cover the vertices of G?
- **Bounded Diameter ($r = 5, \alpha = 1$):** Given a 5-coloring of K_n, does there exists a set of at most four monochromatic subgraphs of bounded diameter that cover the vertices of K_n?
- **Partitions ($r = 4, \alpha = 1$):** Given a 4-coloring of K_n, does there exists a set of at most four monochromatic connected subgraphs that partition the vertices of K_n?

Open Problems: Lower Bounds

- **Bounded Diameter ($r = 3, \alpha = 1$):** Does there exist a 3-coloring of K_n that cannot be covered with a set of at most two monochromatic subgraph of diameter at most 3?



- **Bounded Diameter ($r = 4, \alpha = 1$):** Does there exist a 4-coloring of K_n that cannot be covered with a set of at most three monochromatic subgraphs of diameter at most 2?



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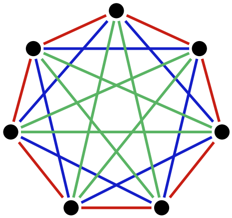

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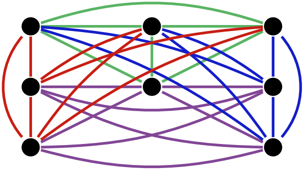

Thanks For Listening